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CALCULATING A YEARLY LIMIT DEPTH TO THE ACTIVE BEACH PROFILE.(U)
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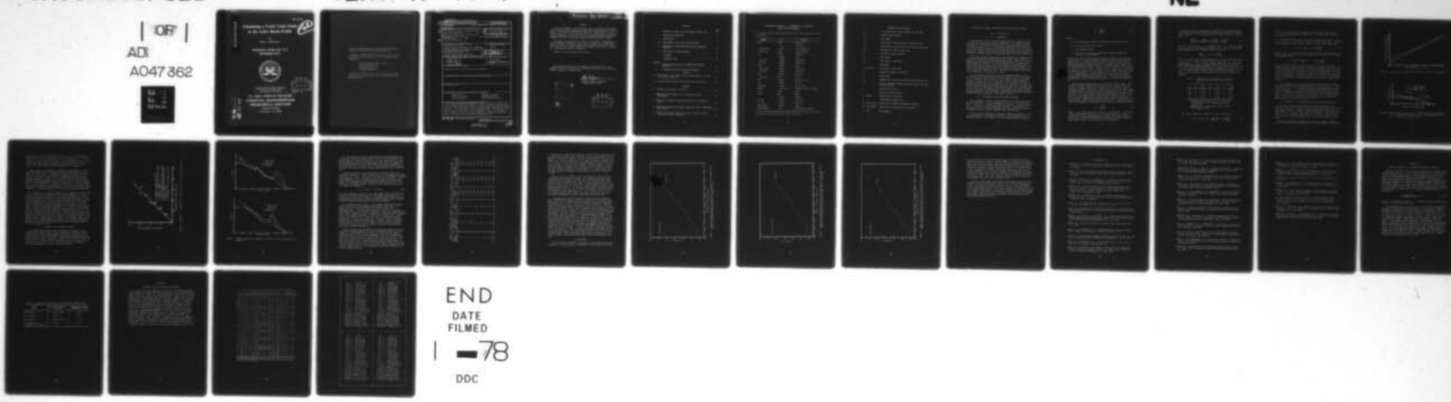

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Calculating a Yearly Limit Depth to the Active Beach Profile

by

Robert J. Hallermeier

TECHNICAL PAPER NO. 77-9

SEPTEMBER 1977

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U.S. ARMY, CORPS OF ENGINEERS
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REPORT DOCUMENTATION PAGE

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1. REPORT NUMBER: TP-77-9	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): 6 CALCULATING A YEARLY LIMIT DEPTH TO THE ACTIVE BEACH PROFILE.	5. TYPE OF REPORT & PERIOD COVERED: 9 Technical Paper
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): 10 Robert J. Hallermeier	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of the Army, Coastal Engineering Research Center (CERRE-CP), Kingman Building, Fort Belvoir, Virginia 22060	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: D31193
11. CONTROLLING OFFICE NAME AND ADDRESS: Department of the Army, Coastal Engineering Research Center, Kingman Building, Fort Belvoir, Virginia 22060	12. REPORT DATE: 11 Sep[redacted] 1977
	13. NUMBER OF PAGES: 28
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): 12 31 p.	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Beach profiles
Coastal engineering
Coastal processes
Sediment entrainment
Sediment transport
Wave-sediment interaction

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A sediment entrainment parameter is used to calculate the maximum water depth for intense agitation of a sand bed by shoaling waves with given height and period. Calculated depths agree with measured water depths over a terrace cut into a fine sand slope by constant laboratory waves. For high wave conditions expected 12 hours per year on exposed U.S. coasts, the calculated depth is about twice the wave height and agrees with published conclusions on the yearly seaward limit to the active beach profile.

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PREFACE

This report provides coastal engineers with a procedure for calculating a limiting depth to appreciable sand level changes seaward of a beach, using an estimate of extreme waves occurring at the locality. The reported results are from an ongoing study of the seaward limit of effective sediment transport, carried out under the sediment-hydraulic interaction program of the U.S. Army Coastal Engineering Research Center (CERC).

The report was written by Dr. Robert J. Hallermeier, Oceanographer, under the general supervision of Dr. Cyril J. Galvin, Jr., Chief, Coastal Processes Branch. The author appreciates the comments and cooperation of Dr. Galvin, C.B. Chesnutt, and Dr. J.R. Weggel, CERC, and J.T. Jarrett, U.S. Army Engineer District, Wilmington.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

John H Cousins

JOHN H. COUSINS
Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.8532	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.1745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1] To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: $C = (5/9)(F - 32)$. To obtain Kelvin (K) readings, use formula: $K = (5/9)(F - 32) + 273.15$.

SYMBOLS AND DEFINITIONS

A	wave-induced water orbit diameter near the bed
D	sediment grain diameter
d	water depth
d_c	water depth at wave-caused cut into plane slope
d_s	water depth at seaward limit of intense bed agitation
d_t	maximum water depth in wave tank
g	acceleration due to gravity
H	wave height
H_o	wave height in deep water
L	wavelength
$L_o = gT^2/2\pi$	wavelength in deep water
L_t	maximum wavelength in wave tank
T	wave period
U_b	maximum wave-induced horizontal velocity near the bed
γ'	ratio of density difference between sediment and fluid to fluid density
ε	number less than one
ν	kinematic fluid viscosity
$\xi = 2\pi d_s/L$	dimensionless limit depth
π	transcendental number 3.14 ...
$\Phi_e = U_b^2/(\varepsilon g\gamma' d)$	dimensionless sediment entrainment parameter
$\Phi_i = U_b^2/(g\gamma' D)$	dimensionless sediment mobility parameter
$\omega = 2\pi/T$	wave frequency

CALCULATING A YEARLY LIMIT DEPTH TO THE ACTIVE BEACH PROFILE

by
Robert J. Hallermeier

I. INTRODUCTION

Certain coastal zone activities require setting a seaward limit to the very active (littoral) zone of a sand beach; e.g., sediment budget calculations, submarine placement of beach fill, and design of coastal structures such as jetties. In principle, repetitive measurements of waves and bathymetry can define the seaward limit at a site by establishing the water depth beyond which bottom changes caused by wave action are negligible. However, these data are costly and rare, so an estimate of the limit depth must often be obtained from other evidence or experience.

Silvester (1974) discussed physical data showing that extreme surface waves occasionally move bottom sands in water depths up to about 100 meters; i.e., over much of the continental shelves. However, reviewing geomorphological evidence, Dietz and Fairbridge (1968) concluded that waves intensively work fine, nearshore sands into a somewhat concave equilibrium profile extending to a water depth of 10 to 20 meters. This depth is the seaward limit to the active profile, supposedly related to wave climate for a particular site. Offshore of this depth lies a zone of less important sand transport by waves.

For these coastal zone activities, a calculation procedure using known wave climate might provide a useful estimate of this limit depth, in case definitive data are lacking. This report proposes a procedure giving a minimum value for the limit depth to the seasonal onshore-offshore transport cycle on sand beaches. The development is based on dimensional reasoning of the energetics of wave agitation of a sand bed, without detailed examination of sand transport processes. Although somewhat speculative, this approach arrives at a simple calculation procedure giving a limit depth for a shoaling two-dimensional wave with a certain height and period. Calculated depths agree with the published laboratory and field evidence considered in Sections III and IV.

II. DEVELOPMENT OF THE CALCULATION PROCEDURE

This development considers wave energy density in terms of two dimensionless parameters. The first parameter (eq. 1) has documented pertinence to the onset of sediment movement as a bedload. A second parameter (eq. 2), similar in form but numerically smaller, is hypothesized to describe the onset of intense bed agitation, characterized by sediment entrainment into the wave flow beyond a thin near-bed layer.

Silvester (1974) presented the empirical expressions derived by various investigators of the motion threshold on a flat sediment bed. An important factor in each expression is the sediment mobility parameter:

$$\Phi_i = \frac{U_b^2}{(g\gamma' D)} \tag{1}$$

where

U_b = the maximum wave-induced horizontal velocity near the bed

g = the acceleration due to gravity

D = the sediment diameter

γ' = the ratio of the density difference between sediment and fluid to the fluid density

Equation (1) gives the peak near-bottom wave energy per unit sediment grain volume, divided by the energy required to raise an immersed grain against gravity a distance equal to half its diameter (Nayak, 1970). Although the motion threshold cannot be described solely in terms of Φ_i, sediment generally begins to move when this number becomes larger than one (Komar and Miller, 1973). Lofquist (1975) observed that Φ_i measured the intensity of sand motion in his tests with a rippled bed; the motion threshold was near $\Phi_i = 2.4$, the ripple crests became less distinct for $\Phi_i > 10$, approximately, and, for larger Φ_i, vortices of fluid and entrained sediment became increasingly evident, indicating more intense bed agitation. These results show that $\Phi_i \sim 1$ indicates the beginning of sediment mobility by describing energetics in a thin layer near the bed.

After sediment grains begin to move, wavy bed features form. These features are caused by grain flow, but liquid flow of high inertia can scour these features from the bed (Bagnold, 1956). Because a plane (intensely agitated) bed usually occurs in and near the surf zone (Inman and Bagnold, 1963), a criterion for the seaward limit to the active beach might be based on a parameter measuring the energetics of intense bed agitation. Available research results (reviewed briefly in App. A) indicate that grain diameter has a weak influence in intense agitation of fine sand beds, suggesting that another Froude number, like equation (1), be used with a larger length scale replacing D. It is hypothesized that intense bed agitation is described by the sediment entrainment parameter:

$$\Phi_e = \frac{U_b^2}{(\varepsilon g\gamma' d)} \tag{2}$$

where ε is a number less than one and d is water depth. (The choice of the length scale replacing D is discussed in App. A.) When Φ_e reaches one, the wave energy density is sufficient to raise a grain a distance $\varepsilon d/2$, taken to be appreciable fraction of the water depth and much greater than D/2 for fine sands (D<0.4 millimeter).

Equation (2) and a coupled pair of quantitative assumptions permit calculating a maximum water depth for intense bed agitation by shoaling waves. Taking $\Phi_e = 1$ and using linear wave theory, equation (2) may be written as

$$\left(\frac{2\pi d_s}{L}\right) \sinh \left(\frac{2\pi d_s}{L}\right) \cosh \left(\frac{2\pi d_s}{L}\right) = \frac{\pi^2}{\gamma' \varepsilon} \left[\frac{H}{L}\right]^2 \qquad (3)$$

where H is wave height, L is wavelength, and d_s is the limit depth. To assist selecting a reasonable magnitude for ε, the case of maximum wave steepness (eq. 100 in Madsen, 1976) is examined:

$$\left(\frac{H}{L}\right)_{max} = 0.14 \tanh \left(\frac{2\pi d}{L}\right) \qquad (4)$$

Table 1 lists the d_s/L solving equations (3) and (4) for four values of ε with $\gamma' = 1.6$ (quartz sand in freshwater or saltwater). For the present purpose, $\varepsilon = 0.03$ seems the proper magnitude. The limit depth then remains well beyond the breaker depth for a steep wave, and the limit to the active profile generally lies seaward of the breakers. This ε is also large enough that the previous assumption of $\varepsilon d >> D$ can be easily satisfied for fine sands. Based on these considerations, $\Phi_e = 1$ with $\varepsilon = 0.03$ is taken to indicate the limit depth of intense bed agitation.

Table 1. Dimensionless water depth, d_s/L, solving equations (3) and (4) for four values of ε.[1]

ε	d_s/L	H/L	H/d_s
0.10	0.0610	0.0512	0.839[2]
0.06	0.1211	0.0898	0.742[2]
0.03	0.1790	0.1133	0.633
0.01	0.2579	0.1296	0.503

[1]Calculated using Table C-2 in U.S. Army, Corps of Engineers, Coastal Engineering Research Center (1975).

[2]Breaking wave, if bed slope is small (see eq. 102 in Madsen, 1976).

With these assumptions, equation (3) can be rewritten as

$$\xi \sinh^2 \xi \tanh^2 \xi \left(1 + \frac{2\xi}{\sinh 2\xi}\right) = 205.6 \left(\frac{H_o}{L_o}\right)^2 \qquad (5)$$

where $\xi = (2\pi d_g/L)$, H_o is wave height in deep water $(d/L>0.5)$, and $L_o = (gT^2/2\pi)$ is wavelength in deep water. The dashed curve in Figure 1 solves equation (5).

For wave steepness greater than 0.015, the difference between H and H_o is less than $\pm$ 10 percent, according to linear wave theory. Ignoring this shoaling wave height change, equation (5) takes the simpler form:

$$\xi \sinh^2 \xi \tanh \xi = 205.6 \left(\frac{H}{L_o}\right)^2 \tag{6}$$

Table 1 shows ξ remains less than $2\pi(0.179)\approx 9/8$, so Maclaurin expansions for the transcendental functions in equation (6) yield the accurate algebraic approximation

$$\xi^4 + \frac{1}{15}\xi^8 - \frac{4}{189}\xi^{10} - \cdots = 205.6\left(\frac{H}{L_o}\right)^2 \tag{7}$$

where the omitted terms are on the order of $(0.002\ \xi^{12})$. The solid straight line in Figure 1 shows the solution of equation (7) if only the first term on the left-hand side is retained; the dotted curve shows the solution of equation (7) where the other terms are appreciable; the solution of equation (6) lies between the dotted and solid curves. According to this development, the maximum water depth for intense bed agitation may be obtained for a certain wave condition by reading the corresponding $(2\pi d_g/L)$ from Figure 1, and multiplying by $\{(L_o/2\pi) \tanh (2\pi d_g/L)\}$ to obtain d_g.

This development has considered the ideal two-dimensional situation of a monochromatic wave shoaling according to linear wave theory. Grace and Rocheleau (1973) concluded that available field measurements indicate linear wave theory " ... provides an excellent prediction of the sample mean, near-bottom water velocity beneath the crest of long design-type waves...." LeMehaute, Divoky, and Lin (1968) reported laboratory measurements showing linear wave theory accurately gives U_b in near-breaking waves. Thus, linear wave theory is a convenient and accurate first approximation in the problem considered.

III. COMPARISON OF CALCULATED RESULTS WITH LABORATORY PROFILES

The results from a laboratory test of profile development with a constant wave condition incident on an initially plane slope of fine sand are shown in Figure 2. Changes in profile shape become minor after a large number of waves, and many "equilibrium profiles" have been published for various test conditions (see App. B). Frequently, these profiles exhibit a long, slightly sloping terrace (as in Fig. 2), caused by offshore deposition of sand eroded from the initial slope above a certain depth.

Raman and Earattupuzha (1972) pointed out the existence of stable points occurring in wave development of a sand bed profile. If the waves

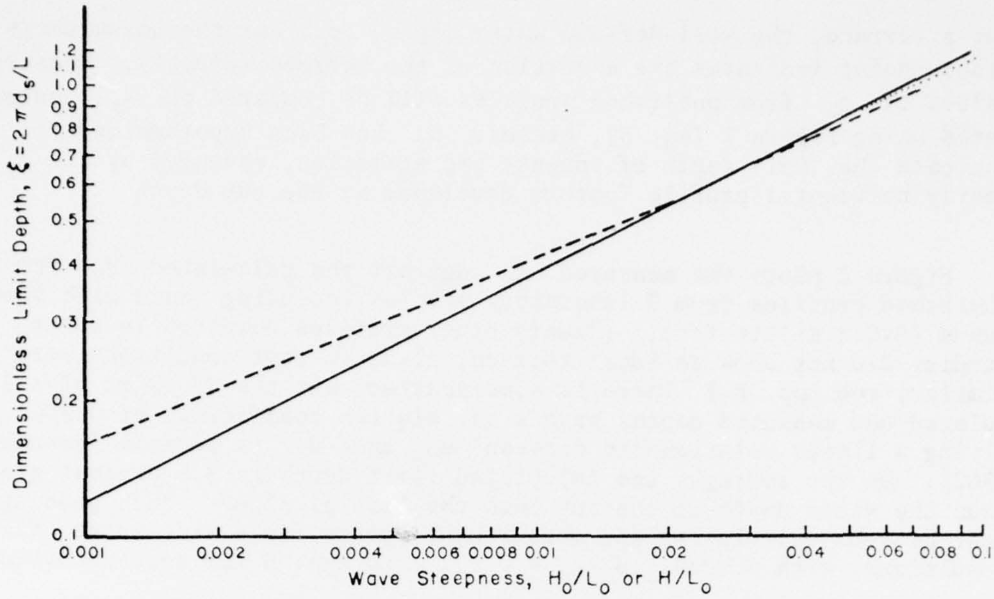

Figure 1. Solutions of equations (5) (dashed curve) and (7) (dotted curve).

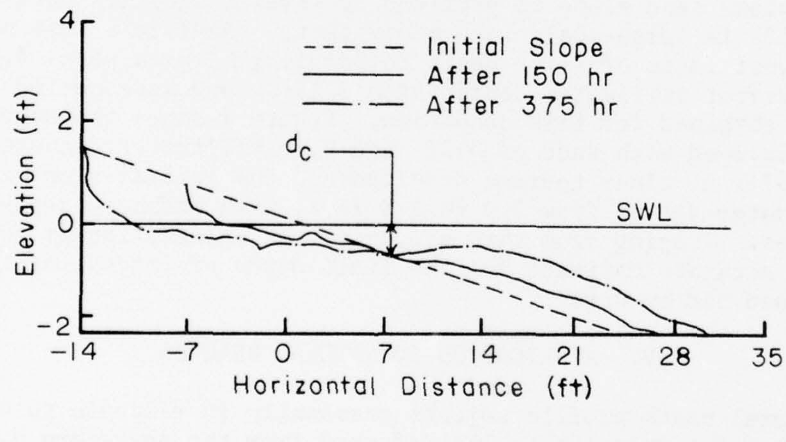

Figure 2. Definition of cut depth, d_c, in laboratory profile development; T = 1.9 seconds, H = 0.36 foot, d_c = 0.73 foot (profiles from Fig. 7 in Chesnutt and Galvin, 1974).

cut a terrace, the well-defined water depth, d_c, at the seawardmost stable point indicates the elevation of the terrace (Fig. 2). Measured values of d_c from published profiles will be compared to d_s, calculated using Figure 1 (eq. 5), because d_s has been hypothesized to indicate the limit depth of intense bed agitation, revealed by the nearly horizontal profile feature developed at the cut depth.

Figure 3 plots the measured d_c against the calculated d_s for 27 published profiles from 7 laboratory studies including tests with fine sands ($D<0.4$ millimeter). (Twenty other profiles obtained in these studies did not show an ideal terrace, although test conditions were similar; see App. B.) There is some scatter, but the 27 pairs of calculated and measured depths have a correlation coefficient of 0.936, implying a linear relationship between d_c and d_s is certain (Freund, 1962). On the average, the calculated limit depth is 3.6 percent greater than the water depth at the cut into the initial slope. This good agreement between calculation and measurement occurs for a wide range of wave conditions, with $0.0039 \leq H_o/L_o \leq 0.071$ (see App. B for test conditions).

The reduced scale of these laboratory tests, compared to natural prototypes, should not be important for the present comparison, because the threshold of intense bed agitation is evidently crossed in the terrace zone of the laboratory tests. Also, the calculation procedure is based on wave energy per unit sediment grain volume, and fine sand occurs both in the laboratory tests and in the offshore region of natural beaches. Limited verification of the unimportance of scale effects on the cut depth in a plane sand slope is provided by several profiles obtained by Saville (1957) in large-scale laboratory tests. Saville's test number 5 had the largest ratio of water depth to wavelength, with waves 4.8 feet high in a 15-foot stillwater depth and a 3.75-second wave period; d_s = 8.3 feet is obtained for this condition. Figure 4 shows the unpublished profiles developed with sand of 0.22- and 0.46-millimeter diameter. Although there is no clear terrace development, the initial slope is eroded onshore of water depths from 7.0 to 8.8 feet, with offshore deposition in both cases. Judging from this evidence, the calculation procedure provides an accurate estimate for the limit depth of intense agitation of a fine sand bed by waves.

IV. APPLICATION TO NATURAL BEACHES

The natural beach profile adjusts seasonally in response to wave climate with relatively little loss of sand from the nearshore over an average yearly cycle. Subaerial profile erosion by storm waves can be notable, but most eroded sand remains in relatively shallow water and is carried onshore by less steep waves. Bruun (1954) and Winant, Inman, and Nordstrom (1975) analyzed southern California data showing that the submarine profile beyond the 5- to 10-meter water depth is accurately described throughout the year by a constant function.

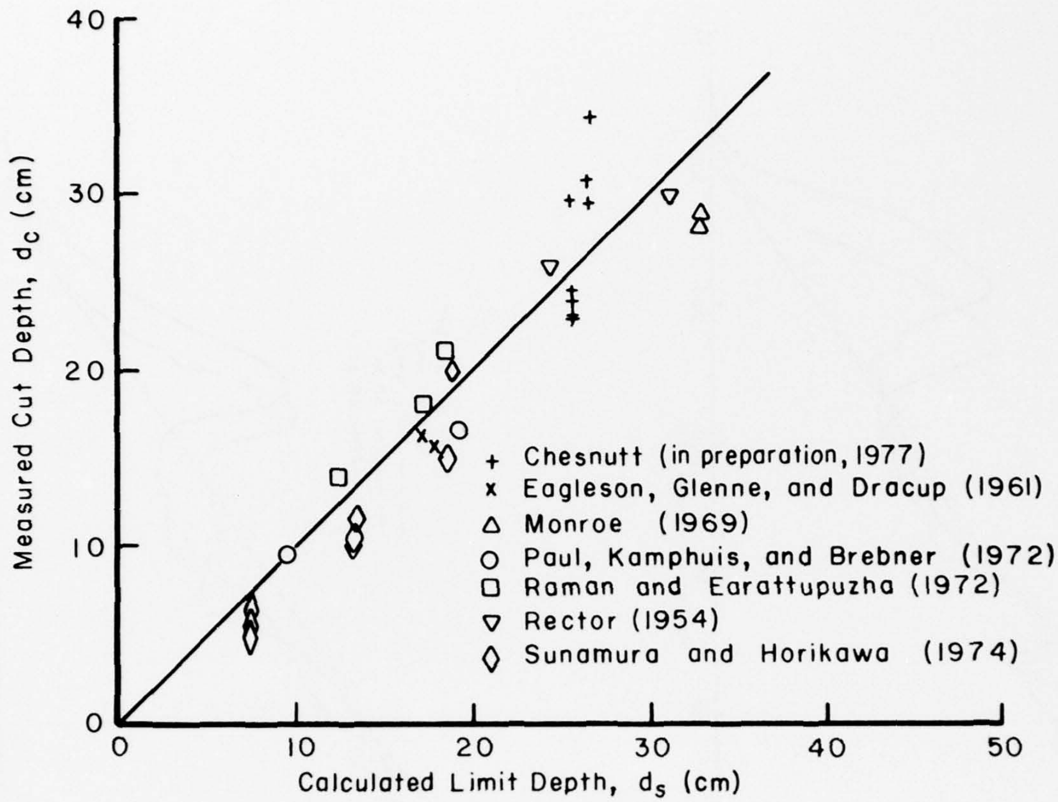

Figure 3. Measured d_c compared with calculated d_s for 27 laboratory tests.

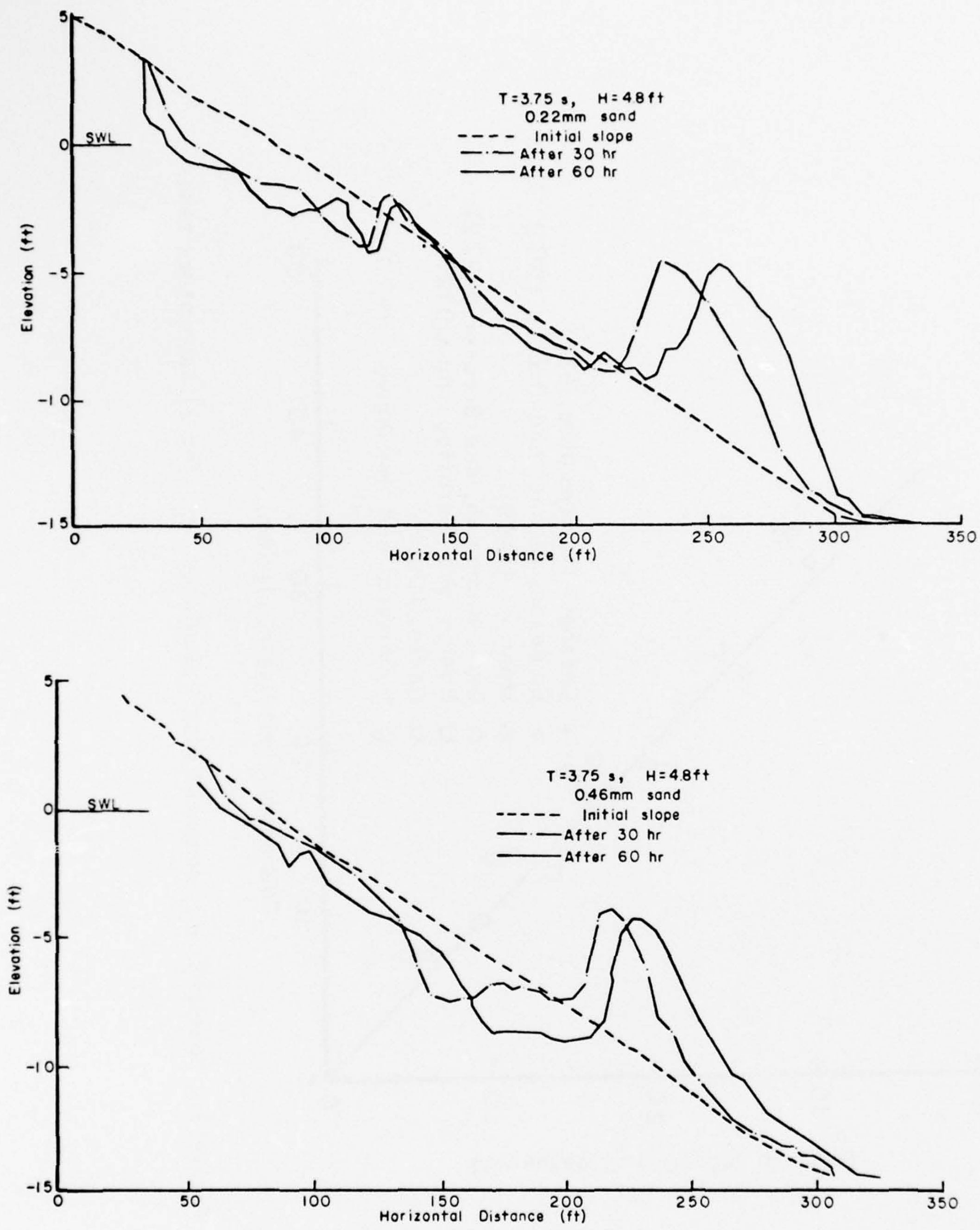

Figure 4. Unpublished profiles developed in Saville's (1957) large-scale test number 5.

The limit depth of interest is the offshore edge of the region participating in the yearly onshore-offshore cycle of sand transport. With the rationale that extreme storm waves will carry sand beyond the limit depth, removing it from the cycling system, this yearly limit depth can be estimated using the previous development with a yearly extreme wave condition typical of a specific site. A representative extreme might be the high waves expected to give extreme bottom velocity for 12 hours per year.

Considering nearshore wave measurements in Thompson (1977), the 10 design wave conditions in Table 2 were selected as a representative range for exposed U.S. coasts. The table shows that the limit depths calculated, using equation (6), are all roughly twice the input wave height. This direct dependence on wave height and the fine trends of the calculated results are in agreement with an approximate form of equation (6). Retaining only the dominant first term on the left-hand side of equation (7) leads to the expression:

$$d_s \simeq 2.28\ H\ (1 - 4.78\ H/L_o) \qquad (8)$$

For the 10 conditions in Table 2, at a given wave height, the calculated limit depth increases above twice the wave height as wave period or L_o increases; at a given wave period, the calculated depth drops below twice the decreasing wave height. Equation (8) gives a depth within 3 percent of that obtained using equation (6) for these conditions.

The calculated results (Table 2) show general agreement with other published conclusions concerning the limit depth to the active beach (Bruun, 1954; Dietz and Fairbridge, 1968; Winant, Inman, and Nordstrom, 1975). Some unpublished bathymetry studies at the Coastal Engineering Research Center (CERC) resulted in conclusions summarized by Duane (1976). Repetitive profiles superposed, revealing little bed change beyond these approximate depths: 15 feet for the Great Lakes, 20 feet for the U.S. Atlantic coast, and 25 feet for the southern California Pacific coast. Design wave conditions 2, 5, and 8 in Table 2 would be reasonable first choices for these three locations, and the calculated depths agree with the stated limit depths, so it appears the proposed calculation procedure may give an accurate estimate of the yearly seaward limit to the active profile.

Further evaluation of the proposed calculation procedure can be made using bottom changes offshore of La Jolla, California, which have been intensively investigated by the Scripps Institution of Oceanography. Shepard and Inman (1951) reported bathymetric survey data establishing significant bed cut and fill to water depths of at least 100 feet; however, "On the outer shelf the movement takes place more parallel to the shore than normal to it." Inman and Rusnak (1956) measured ranges in sand level over a 3-year period. In mean water depths of 18, 30, 52, and 70 feet, ranges were (>2), 0.29, 0.16, and 0.15 foot, respectively, and a seasonal trend in sand level was detected at the 30-foot depth, but not at the two deeper stations.

Table 2. Calculated limit depths for 10 design wave conditions (1 foot = 0.305 meter).

Wave condition	Significant wave height (ft)	Wave period (s)	Steepness, H/L_o	Depth from eq. (6) (ft)	Depth from eq. (8) (ft)
1	6	6	0.0325	11.55	11.55
2	8	6	0.0434	14.82	14.45
3	8	8	0.0244	15.91	16.11
4	10	8	0.0305	19.42	19.47
5	10	10	0.0195	20.29	20.67
6	12	10	0.0234	23.92	24.29
7	16	10	0.0313	31.09	31.03
8	12	12	0.0163	24.96	25.23
9	16	12	0.0217	31.97	32.70
10	20	12	0.0271	39.55	39.69

Nordstrom and Inman (1975) conducted a study of wave climate and beach response at Torrey Pines Beach, north of La Jolla; beach profiles were measured at approximate monthly intervals during June 1972 to April 1974. Pawka, et al. (1976) reported the wave climate recorded by offshore pressure sensors up to four times daily from February 1973 to May 1974. During the study, Torrey Pines Beach underwent seasonal changes in configuration related to changes in wave regime. High-energy winter waves removed sand from the subaerial beach and deposited it offshore at depths less than 30 feet below mean sea level (MSL). Low-energy summer waves returned sand to the subaerial beach from depths less than 20 feet below MSL.

Figure 5 shows the offshore survey data along the three ranges during the Torrey Pines Beach study (App. B in Nordstrom and Inman, 1975). Measured sand level changes were less than 2 feet beyond a water depth of about 21 feet relative to MSL, and less than 1 foot beyond a water depth of about 26 feet relative to MSL. These fathometer profile surveys have a stated accuracy of ± 1 foot, so recorded changes less than 1 foot indicate profile closure. The reference rod arrays at 24 feet below MSL showed ranges in sand level of 1.2 to 1.5 feet during the study; arrays at 33 feet below MSL showed ranges of 0.2 to 0.4 foot, with a stated measurement accuracy of 0.3 foot. These data show that 26 feet below MSL is the limit depth to the active profile at the study site.

A characteristic yearly design wave should be chosen considering both the breaker observations tabulated in Appendix D of Nordstrom and Inman (1975), and the offshore wave measurements tabulated in Appendix D of Pawka, et al. (1976). Among the 470 daily observations during 23 months, 10-foot-high breakers were noted three times, with a 14-second period on 12 February 1973, an 18-second period on 28 February 1973, and a 10-second period on 18 April 1973. Among the 657 wave measurements during 16 months, the most energetic condition was reported as 3,840 square centimeters, centered at a period of 14.2 seconds on 12 February 1973; this energy level corresponds to a significant deepwater height of 7.2 feet, and a significant height of 8.0 feet in a 33-foot depth. The density of either wave data set is not sufficient to make the extreme tabulated condition represent a 12-hour per year duration, so a higher wave should be chosen as the design condition. A nearshore height of 10 to 11 feet and a period of 10 to 18 seconds seem appropriate design wave conditions for this locality. For these conditions, equation (6) gives depths of 20.3 to 23.9 feet. At Torrey Pines Beach, mean lower low water is -2.7 feet relative to MSL; adding this possible tidal effect to the calculated depth results in a limit depth estimate of 23.0 to 26.6 feet relative to MSL. Thus, the proposed calculation procedure gives an accurate estimate of the actual limit depth to the active profile in this case.

V. CONCLUSION

Using the number introduced in equation (2) and several quantitative assumptions, the water depth at a hypothetical seaward limit of intense

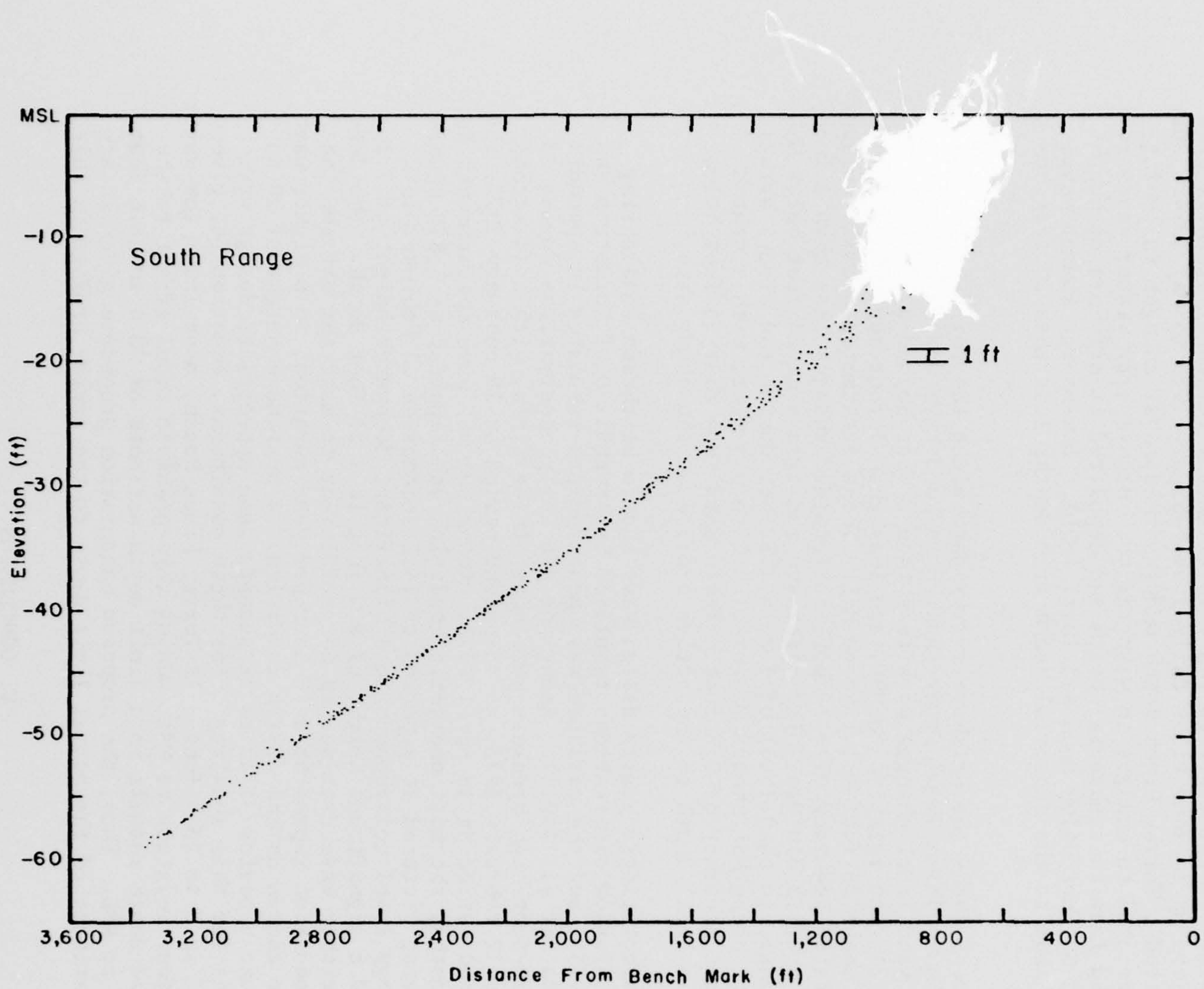

Figure 5. Offshore survey data from 6 June 1972 to 30 April 1974 at Torrey Pines Beach, California (from Nordstrom and Inman, 1975).

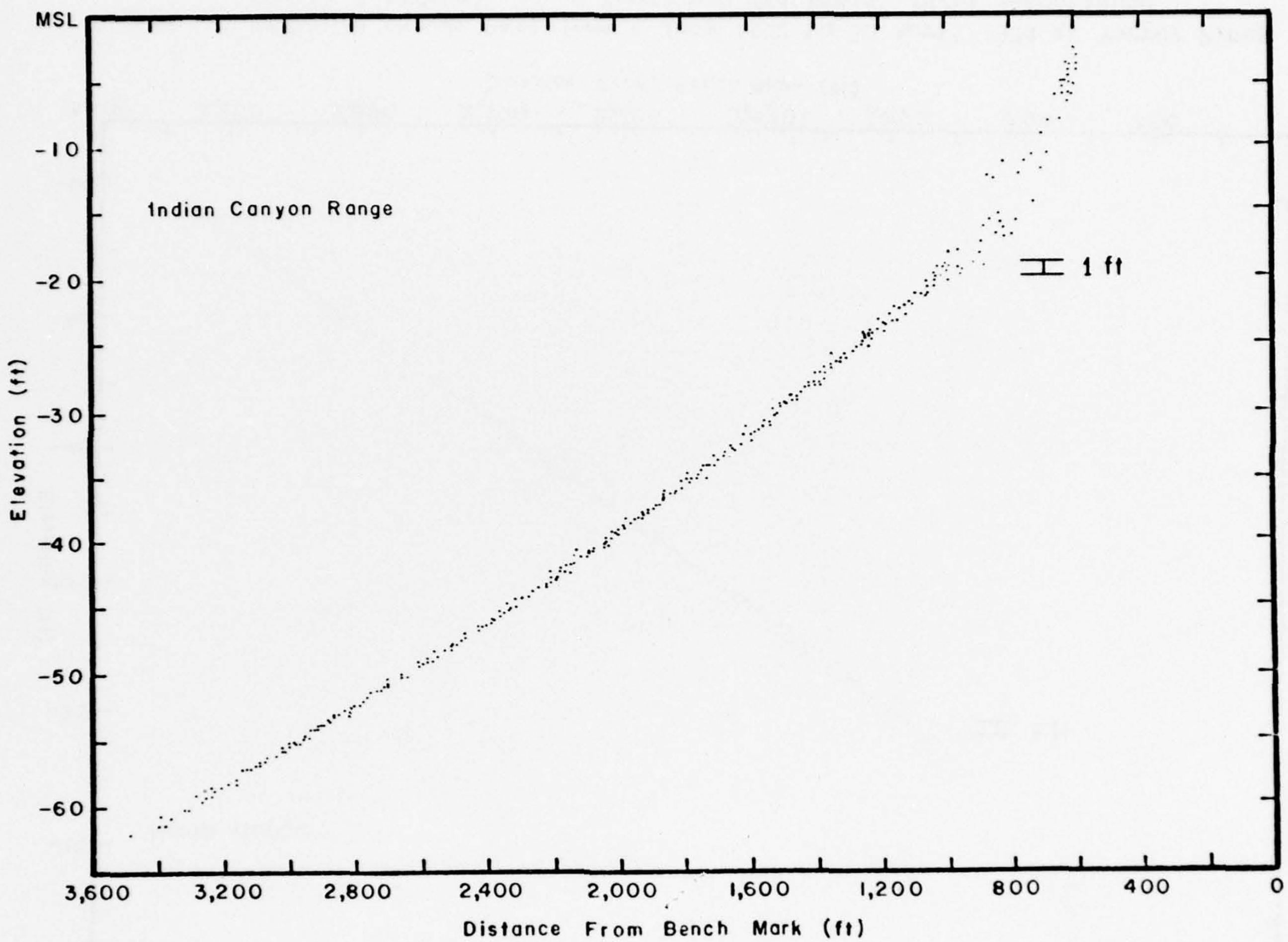

Figure 5. Offshore survey data from 6 June 1972 to 30 April 1974 at Torrey Pines Beach, California (from Nordstrom and Inman, 1975).--Continued

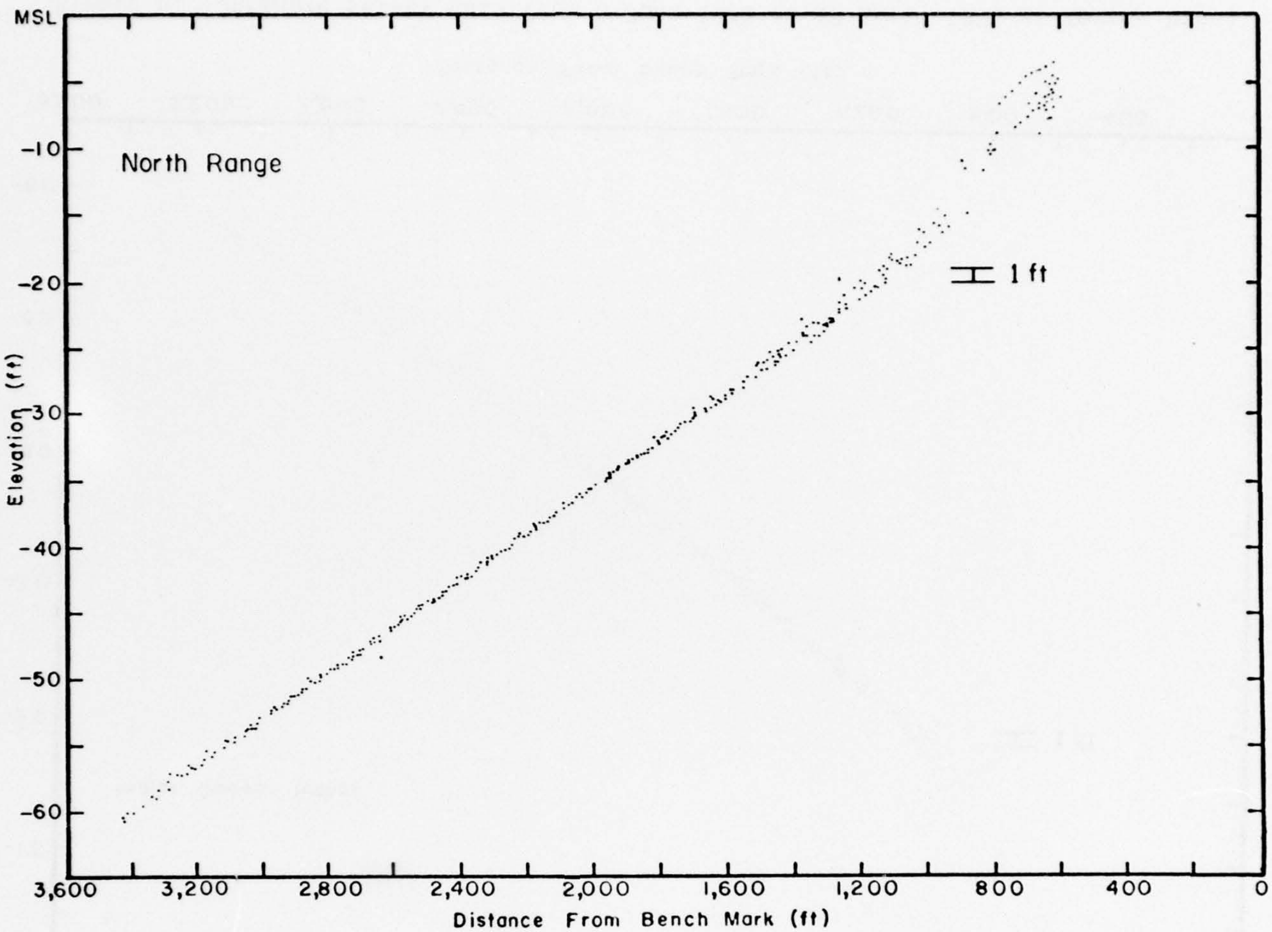

Figure 5. Offshore survey data from 6 June 1972 to 30 April 1974 at Torrey Pines Beach, California (from Nordstrom and Inman, 1975).--Continued

bed agitation can be calculated using Figure 1 or equation (8) with a given wave height and period. Factors ignored in the presented development, such as tidal currents, sand size, and bottom slope, can clearly be pertinent to the limit depth of the active beach, but wave height and period have long been recognized as dominant factors in beach processes. Crucial and somewhat arbitrary choices are taking $\Phi_e = 1$ and $\varepsilon = 0.03$ in using equation (2). Since the calculated limit depths for natural beaches are about 30 feet, $(\varepsilon d_g/2)$ is about 0.5 foot and the wave energy can raise bed sand above large, natural ripples with heights up to 0.5 foot (Inman, 1957). Thus, the criterion used for intense bed agitation is internally consistent with calculated results.

Calculated depths agree with measured water depths over the terrace frequently cut into plane slope of fine sand by constant laboratory wave action. For high waves expected 12 hours per year, calculated depths agree with previously published conclusions on the yearly limit depth to the active beach profile. With a more extreme wave condition as input, the calculation procedure could provide an estimate of the seaward limit over a longer period; however, field data are not available to check such an estimate. The present results encourage further evaluation of the proposed calculation procedure.

LITERATURE CITED

BAGNOLD, R.A., "The Flow of Cohesionless Grains in Fluids", *Philosophical Transactions of the Royal Society of London*, Ser. A, Vol. 249, 1956, pp. 235-297.

BRUUN, P., "Coast Erosion and the Development of Beach Profiles," TM-44, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., June 1954.

CHAN, K.W., BAIRD, M.H.I., and ROUND, G.F., "Behaviour of Beds of Dense Particles in a Horizontally Oscillating Fluid," *Proceedings of the Royal Society of London*, Vol. A 330, 1972, pp. 537-559.

CHESNUTT, C.B., "Analysis of Results from 10 Movable-Bed Experiments," Vol. VIII, MR 77-7, *Laboratory Effects in Beach Studies*, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., in preparation, 1977.

CHESNUTT, C.B., and GALVIN, C.J., Jr., "Lab Profile and Reflection Changes for $H_o/L_o = 0.02$," *Proceedings of the 14th Coastal Engineering Conference*, 1974, pp. 958-977.

DIETZ, R.S., and FAIRBRIDGE, R.W., "Wave Base," *The Encyclopedia of Geomorphology*, Reinhold, New York, 1968, pp. 1224-1228.

DUANE, D.B., "Sedimentation and Coastal Engineering: Beaches and Harbors," *Marine Sediment Transport and Environmental Management*, Wiley, New York, 1976, pp. 493-517.

EAGLESON, P.S., GLENNE, B., and DRACUP, J.A., "Equilibrium Characteristics of Sand Beaches in the Offshore Zone," TM-126, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., July 1961.

FREUND, J.E., *Mathematical Statistics*, Prentice-Hall, Englewood Cliffs, N.J., 1962.

GRACE, R.A., and ROCHELEAU, R.Y., "Near-Bottom Velocities under Waikiki Swell," Rep. 73-31, University of Hawaii, Honolulu, 1973.

INMAN, D.L., "Wave-Generated Ripples in Nearshore Sands," TM-100, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., Oct. 1957.

INMAN, D.L., and BAGNOLD, R.A., "Littoral Processes," *The Sea*, Vol. 3, Interscience, New York, 1963, pp. 523-549.

INMAN, D.L., and RUSNAK, G.S. "Changes in Sand Level on the Beach and Shelf at La Jolla, California," TM-82, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., Sept. 1956.

KOMAR, P.D., and MILLER, M.C., "The Threshold of Sediment Movement under Oscillatory Water Waves," *Journal of Sedimentary Petrology*, Vol. 43, No. 4, Dec. 1973, pp. 1101-1110.

LeMEHAUTE, B., DIVOKY, D., and LIN, A., "Shallow-Water Waves: A Comparison of Theories and Experiments," *Proceedings of the 11th Coastal Engineering Conference*, 1968, pp. 86-107.

LOFQUIST, K.E.B., "An Effect of Permeability on Sand Transport by Waves," TM-62, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Dec. 1975.

MADSEN, O.S., "Wave Climate of the Continental Margin: Elements of its Mathematical Description," *Marine Sediment Transport and Environmental Management*, Wiley, New York, 1976, pp. 65-87.

MOGRIDGE, G.R., and KAMPHUIS, J.W., "Experiments on Bed Form Generation by Waves," *Proceedings of the 13th Coastal Engineering Conference*, 1972, pp. 1123-1142.

MONROE, F.F., "Oolitic Aragonite and Quartz Sand: Laboratory Comparison under Wave Action," MP 1-69, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Washington, D.C., Apr. 1969.

NAYAK, I.V., "Equilibrium Profiles of Model Beaches," Report HEL 2-25, Hydraulic Engineering Laboratory, University of California, Berkeley, Calif., 1970.

NORDSTROM, C.E., and INMAN, D.L., "Sand-Level Changes on Torrey Pines Beach, California," MP 11-75, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Dec. 1975.

PAUL, M.J., KAMPHUIS, J.W., and BREBNER, A., "Similarity of Equilibrium Beach Profiles," *Proceedings of the 13th Coastal Engineering Conference*, 1972, pp. 1217-1236.

PAWKA, S.S., et al., "Wave Climate at Torrey Pines Beach, California," TP 76-5, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., May 1976.

RAMAN, H., and EARATTUPUZHA, J.J., "Equilibrium Conditions in Beach Wave Interaction," *Proceedings of the 13th Coastal Engineering Conference*, 1972, pp. 1237-1256.

RECTOR, R.L., "Laboratory Study of Equilibrium Beach Profiles," TM-41, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., Aug. 1954.

SAVILLE, T., Jr., "Scale Effects in Two Dimensional Beach Studies," *Proceedings of the Seventh General Meeting, International Association of Hydraulic Research*, 1957, pp. A3-1 - A3-10.

SHEPARD, F.P., and INMAN, D.L., "Sand Movement on the Shallow Inter-Canyon Shelf at La Jolla, California," TM-26, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., Nov. 1951.

SILVESTER, R., *Coastal Engineering*, Vol. 2, Elsevier, Amsterdam, 1974, pp. 33-61.

SUNAMURA, T., and HORIKAWA, T., "Two-Dimensional Beach Changes due to Waves," *Proceedings of the 14th Coastal Engineering Conference*, 1974, pp. 920-938.

THOMPSON, E.F., "Wave Climate at Selected Locations along U.S. Coasts," TR 77-1, U.S. Army, Corps of Engineers, Coastal Engineering Research Center, Fort Belvoir, Va., Jan. 1977.

U.S. ARMY, CORPS OF ENGINEERS, COASTAL ENGINEERING RESEARCH CENTER, *Shore Protection Manual*, 2d ed., Vols. I, II, and III, Stock No. 008-022-00077-1, U.S. Government Printing Office, Washington, D.C., 1975, 1,160 pp.

WATTS, G.M., "Laboratory Study of Effect of Varying Wave Periods on Beach Profiles," TM-53, U.S. Army, Corps of Engineers, Beach Erosion Board, Washington, D.C., Sept. 1954.

WINANT, C.D., INMAN, D.L., and NORDSTROM, C.E., "Description of Seasonal Beach Changes using Empirical Eigenfunctions," *Journal of Geophysical Research*, Vol. 80, No. 15, May 1975, pp. 1979-1986.

APPENDIX A

CHOICE OF LENGTH SCALE IN SEDIMENT ENTRAINMENT PARAMETER

Published results for oscillatory flow over horizontal sediment beds indicate grain diameter, D, has a weak influence on the transition from a gently agitated, rippled bed to an intensely agitated bed with lesser bed forms. Mogridge and Kamphuis (1972) reported wave tank and water tunnel tests showing that bed-form height reaches a maximum and then steadily decreases with increasing A/D, where A is near-bottom horizontal fluid orbit diameter; the maximum bed-form height depends primarily on A and only slightly on D. Chan, Baird, and Round (1972) reported tests in a small-diameter pipe showing a well-defined transition from a rippled bed to a regime of intense bed agitation with surface particles in motion throughout the oscillation cycle. The empirical expression for this transition is

$$\frac{A\ \omega^{1.2}}{(\gamma' g)^{0.5}\ D^{0.1}\ \nu^{0.2}} = 6.6, \qquad \text{(A1)}$$

where ω is oscillation frequency, ν is kinematic fluid viscosity, and D occurs as a relatively weak factor.

The sediment mobility parameter (eq. 1) is a function of wave height and period, water depth, and grain diameter. In writing the sediment entrainment parameter (eq. 2), replacing D by εd constitutes a direct approach to clarifying the algebraic dependence of the limit depth, d_s, on wave height and period, combined in (H_o/L_o). Because definitive data on sediment entrainment are lacking, any choice for the length scale in the denominator of this Froude number must be tentative on physical grounds. Four lengths naturally occur in a shoaling wave: H, d, L, and A. For the four possible expressions, Table A-1 shows the resulting functional dependence of limit depth on wave steepness (corresponding to eq. 6), and the value of each expression at $\xi = 0.9$ divided by its value at $\xi = 0.3$, the important range of limit depth. Using εd for the length scale gives calculated results in agreement with experimental evidence over a wide range of ξ or H_o/L_o (Fig. 3 and App. B), so the other possible choices cannot, because the other values in the last column of Table A-1 differ greatly from the first.

Table A-1. Comparison of results with four possible length scales.

Length scale	Resulting dependence on $(H/L_o)^2$	$\left(\frac{\text{Value at } \xi = 0.9}{\text{Value at } \xi = 0.3}\right)$
Water depth, d	$\xi \sinh^2 \xi \tanh \xi$	83.82
Wave height, H	$\sinh^4 \xi$	129.12
Wavelength, L	$\sinh^2 \xi \tanh \xi$	27.94
Near-bottom orbit Diameter, $A = H/\sinh \xi$	$\sinh^2 \xi$	11.37

APPENDIX B

LABORATORY STUDIES OF PROFILE DEVELOPMENT

The data base for the comparison in Figure 3 is 47 published profiles from 7 laboratory studies with constant wave action on an initially plane slope of fine sand. Table B-1 lists these profiles from seven references. Column 1 gives the original test identification. Column 2 lists the slope of the initial plane beach; values in parentheses indicate the slope was truncated by a steeper offshore slope. Column 3 gives the sand diameter, and column 4 gives the time elapsed in forming the profile. Columns 5 and 6 list the wave condition; H_o/L_o was calculated using linear wave theory and the reported condition, where necessary. Column 7 lists the maximum water depth in the test tank in the dimensionless form d_t/L_t, where L_t is the maximum wavelength in the tank. Column 8 gives the limit depth calculated from equation (5) for the listed H_o/L_o; value in parentheses indicates the calculated depth is equal to or greater than the column 7 depth. Column 9 gives the calculated limit depth, d_s, unless parentheses occur in column 8. Column 10 gives the cut depth, d_c; value in parentheses indicates the cut depth is uncertain, and a dash indicates no clear cut occurred. Column 11 gives the percentage difference between d_s and d_c; the 27 values not in parentheses are the tests included in Figure 3. Column 12 lists subjective profile classification.

Table B-1. Test conditions and final profile characteristics in seven laboratory studies.

1. Test I.D.	2. Initial slope	3. Sand diameter, (mm)	4. Running time, (hr)	5. Wave condition H_o/L_o	6. Wave condition T, (s)	7.[1] Water depth d_t/L_t	8. Equation (5) Limit depth, d_g/L	9. d_g, (cm)	10. d_σ, (cm)	11. $\frac{200(d_g - d_\sigma)}{(d_g + d_\sigma)}$ (pct)	12. Final profile class[2]
[Chesnutt, 1977]											
70X-06	1/10	0.23	210	0.0211	1.9	0.163	0.089	25.51	23.2	9.5	a
70X-10	1/10	0.23	175	0.0211	1.9	0.163	0.089	25.51	23.2	9.5	a
71Y-06	1/10	0.23	375	0.0211	1.9	0.163	0.089	25.51	23.8	6.9	a
71Y-10	1/10	0.23	335	0.0211	1.9	0.163	0.089	25.51	24.4	4.4	a
72A-06	1/10	0.23	135	0.0039	3.75	0.074	0.044	26.51	30.5	-14.0	b,e
72A-10	1/10	0.23	80	0.0039	3.75	0.074	0.044	26.51	34.7	-26.8	b,e
72B-06	1/10	0.23	150	0.0127	2.35	0.125	0.072	26.59	(60.6)	(-78.0)	c
72B-10	1/10	0.23	150	0.0127	2.35	0.125	0.072	26.59	(25.6)	(3.8)	e,f
72C-10	1/10	0.23	140	0.0391	1.5	0.227	0.119	26.61	29.7	-14.0	a,e
72D-06	1/20	0.23	180	0.0211	1.9	0.163	0.089	25.51	29.8	-15.5	a
[Eagleson, Glenne, and Dracup, 1961]											
1	1/30	0.37	121	0.030	1.53	0.160	0.104	21.85	(18.2)	(18.2)	d
2	1/30	0.37	167	0.046	1.15	0.258	0.125	17.01	16.4	3.7	a
3	1/20	0.37	137	0.048	1.15	0.253	0.128	17.55	16.0	9.2	a
4	1/45	0.37	222	0.025	1.53	0.162	0.095	18.55	(14.4)	(25.2)	d
[Monroe, 1969]											
1	1/15	0.28	35	0.075	1.19	0.231	0.156	25.96	------	------	f
2	1/15	0.28	47	0.035	1.77	0.133	0.111	32.92	28.0	16.1	a
3	1/15	0.28	16	0.008	3.58	0.060	(0.060)	-----	------	------	f
4	1/15	0.28	41	0.003	3.58	0.062	0.037	16.87	------	------	f
5	1/15	0.28	35	0.013	3.58	0.062	(0.073)	-----	------	------	f
6	1/15	0.28	47	0.001	5.06	0.043	0.030	22.49	------	------	f
7	1/15	0.28	47	0.004	5.06	0.039	(0.044)	---	------	------	g
8	1/15	0.28	79	0.035	1.77	0.132	0.111	32.92	28.8	8.8	a
[Paul, Kamphuis, and Brebner, 1972]											
Series 5	1/10	0.357	>36	0.016	1.29	0.183	0.080	9.53	9.5	0.3	a,c
"	1/10	0.357	>36	0.020	1.29	0.183	0.087	11.22	------	------	f
"	1/10	0.357	>36	0.025	1.29	0.183	0.095	13.36	------	------	f
"	1/10	0.357	>36	0.030	1.29	0.183	0.104	15.49	------	------	f
"	1/10	0.357	>36	0.040	1.29	0.183	0.118	19.29	16.7	14.4	a,c
[Raman and Earattupuzha, 1972]											
1	1/8	0.3	40	0.043	1.0	0.246	0.122	12.24	13.9	-12.7	a,c
2	1/10	0.3	42	0.043	1.0	0.246	0.122	12.24	(13.8)	(-12.0)	f
3	1/12	0.3	43	0.071	1.0	0.246	0.153	17.04	18.0	-5.5	a,c
4	1/15	0.3	42	0.042	1.0	0.246	0.120	11.88	(17.0)	(-35.5)	f
5	1/10	0.3	45	0.012	2.0	0.100	0.071	18.48	21.1	-13.2	a
[Rector, 1954; Watts, 1954]											
7,1	(1/30)	0.22	50	0.006	3.3	-----	0.053	28.91	------	------	d
7,2	1/30	0.22	70	0.025	1.75	-----	0.095	24.51	25.9	-5.5	a
7,3	1/30	0.22	150	0.017	2.2	-----	0.081	38.81	------	------	d
7,4	(1/30)	0.22	15	0.010	2.75	-----	0.065	29.92	------	------	d
7,5	(1/30)	0.22	100	0.038	1.3	-----	0.115	18.87	------	------	f
11,2	1/20	0.22	60	0.011	2.68	0.112	0.068	30.70	(16.4)	(-73.5)	f
15,2	1/20	0.22	60	0.024	2.00	0.160	0.094	31.23	29.9	4.4	b
[Sunamura and Horikawa, 1974]											
2	1/10	0.2	160	0.022	1.0	-----	0.090	7.20	6.2	14.9	a
4	1/10	0.2	160	0.079	1.0	-----	0.128	13.39	10.2	27.0	a
6	1/10	0.2	160	0.012	2.0	0.117	0.071	18.69	20.0	-6.8	a
8	1/20	0.2	160	0.022	1.0	-----	0.090	7.20	6.4	11.8	a
10	1/20	0.2	160	0.049	1.0	-----	0.128	13.39	10.7	22.3	a
12	1/20	0.2	160	0.012	2.0	-----	0.071	18.69	15.2	20.6	b
14	1/30	0.2	160	0.022	1.0	-----	0.090	7.20	6.7	7.2	a
16	1/30	0.2	160	0.049	1.0	-----	0.128	13.39	11.6	14.3	a

[1]Water depth not indicated in 12 tests.

[2]Key to profile classes: a - ideal terrace; b - well-defined terrace with slight erosion offshore of toe; c - little profile development; d - offshore slope erosion; e - slightly three-dimensional profile; f - variable offshore without well-defined terrace; g - slope cut almost at tank bottom.

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